

۱۷، ۳، ۵

باسه تقالی

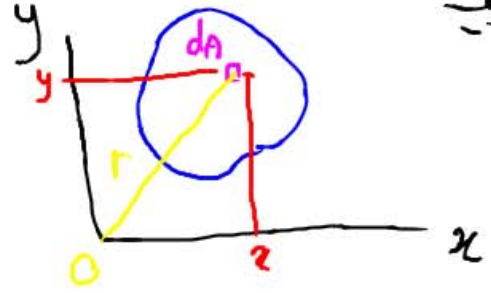
جرم گستاور لختی سطوح

ار تعریف

$$I_x = \int y^2 dA > 0 \quad \begin{matrix} m^4, cm^4 \\ in^4 \end{matrix}$$

$$I_y = \int x^2 dA > 0$$

قطبی  $J_o = \int r^2 dA = I_x + I_y$



$$k_x = \sqrt{\frac{I_x}{A}}$$

$$I_x = k_x^2 A$$

$m, cm$   
 $in, ft$

سُباع عرض

سُباع ژبراسیون

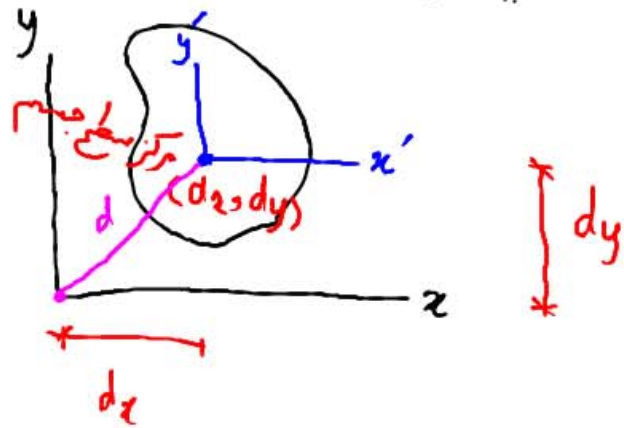
$$k_y = \sqrt{\frac{I_y}{A}}$$

$$k_o = \sqrt{\frac{J_o}{A}}$$

۲، قضیه مورهای موازی

$$I = \bar{I} + A d^2$$

$$k = \bar{k} + d^2$$

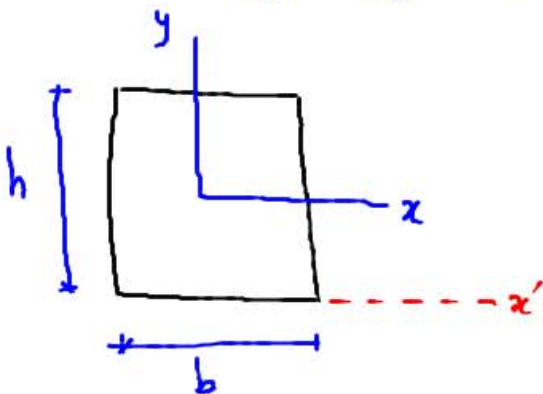


$$I_x = \bar{I}_{x'} + A (d_y)^2$$

$$I_y = \bar{I}_{y'} + A (d_x)^2$$

$$J_o = \bar{J}_{o'} + A d^2$$

مثال



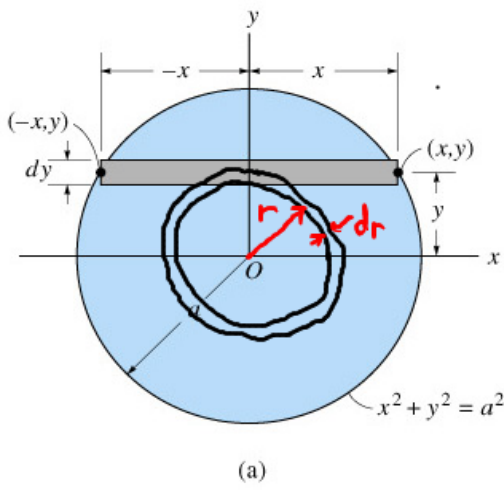
$$I_x = \frac{bh^3}{12}$$

$$I_y = \frac{hb^3}{12}$$

$$I_{x'} = I_x + A (d_y)^2$$

$$I_{x'} = \frac{bh^3}{12} + (bh) \left(\frac{h}{2}\right)^2 = \frac{bh^3}{3}$$

مثال

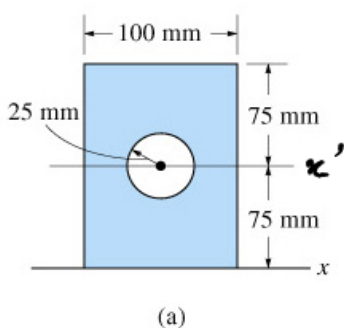


$$I_x = I_y = J_o$$

$$I_x = I_y = \frac{J_o}{2}$$

$$J_o = \int r^2 dA = \int_0^a r^2 (2\pi r dr) = 2\pi \int_0^a r^3 dr = 2\pi \left[ \frac{r^4}{4} \right]_0^a = \frac{2\pi a^4}{4} = \frac{\pi a^4}{2}$$

گسترده تختی سطوح مرکب



مستطیل  $I_x = \frac{100 \times 150^3}{3} = 1.125 \times 10^8 \text{ mm}^4$

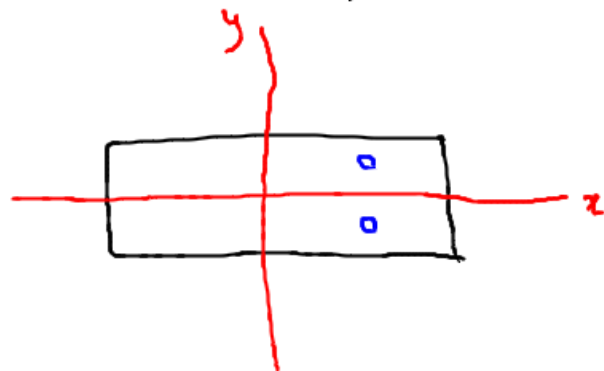
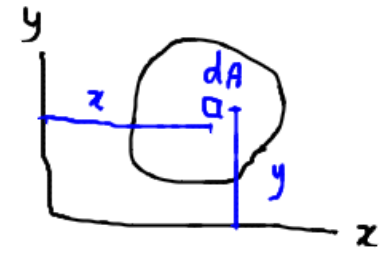
دایره  $\bar{I}_{x'} = \frac{\pi (25)^4}{4}$

دایره  $I_x = \frac{\pi 25^4}{4} + \pi (25)^2 (75)^2 = 1.135 \times 10^7 \text{ mm}^4$

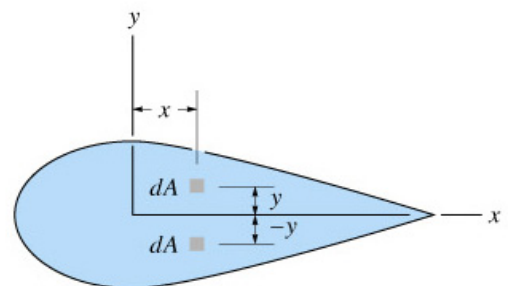
کل  $I_x = I_{\text{مستطیل}} - I_{\text{دایره}} = 1.011 \times 10^8 \text{ mm}^4$

حاصل ضرب تختی

$$I_{xy} = \int xy dA$$



اگر یکی از محورهای x یا y، محور تقارن باشند حاصل ضرب تختی  $I_{xy}$  صفر خواهد بود

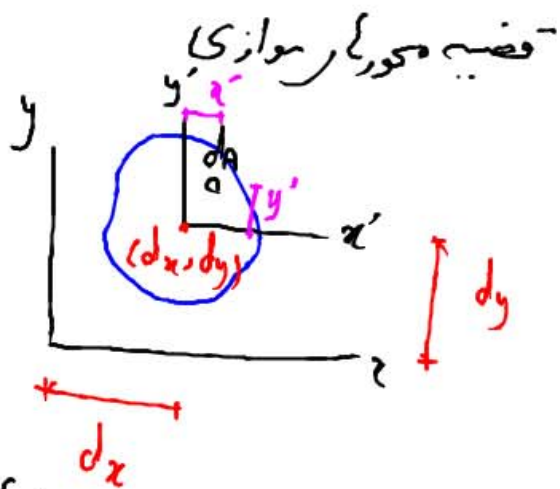


$$I_{xy} = \bar{I}_{x'y'} + A d_x d_y$$

$$I_{xy} = \int (y' + d_y)(x' + d_x) dA$$

$$= \int x'y' dA + d_x \int y' dA +$$

$$+ d_y \int x' dA + d_x d_y \int dA = \bar{I}_{x'y'} + d_x d_y A$$



$$I_x, I_y, I_{xy} \rightarrow I_u, I_v, I_{uv}$$

$$u = x \cos \theta + y \sin \theta$$

$$v = y \cos \theta - x \sin \theta$$

$$I_u = \int v^2 dA$$

$$I_v = \int u^2 dA$$

$$I_{uv} = \int uv dA$$

$$I_u = \int v^2 dA = \int (y \cos \theta - x \sin \theta)^2 dA =$$

$$\int (y^2 \cos^2 \theta + x^2 \sin^2 \theta - 2xy \sin \theta \cos \theta) dA =$$

$$I_u = \cos^2 \theta I_x + \sin^2 \theta I_y - 2 \sin \theta \cos \theta I_{xy}$$

$$\textcircled{1} I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2} \cos 2\theta - I_{xy} \sin 2\theta$$

$$\frac{1 + \cos 2\theta}{2} = \cos^2 \theta$$

$$\textcircled{2} I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2} \cos 2\theta + I_{xy} \sin 2\theta$$

$$\frac{1 - \cos 2\theta}{2} = \sin^2 \theta$$

$$\textcircled{1} I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2} \cos 2\theta + I_{xy} \sin 2\theta$$

$$\frac{I_x - I_y}{2} = \sin^2 \theta$$

$$\textcircled{2} I_{uv} = \frac{I_x - I_y}{2} \sin 2\theta + I_{xy} \cos 2\theta$$

$$I_u - \frac{I_x + I_y}{2} = \frac{I_x - I_y}{2} \cos 2\theta - I_{xy} \sin 2\theta$$

$$I_{uv} = \frac{I_x - I_y}{2} \sin 2\theta + I_{xy} \cos 2\theta$$

$$\left( I_u - \frac{I_x + I_y}{2} \right)^2 + (I_{uv})^2 = \underbrace{\left( \frac{I_x - I_y}{2} \right)^2 + I_{xy}^2}_{R^2}$$

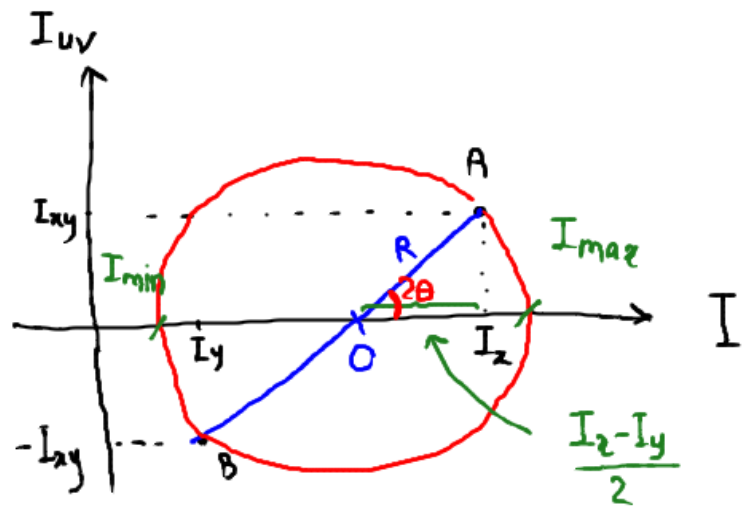
$$\left( I_u - C \right)^2 + I_{uv}^2 = R^2$$

$\frac{I_x + I_y}{2} \quad I_u, I_{uv} \quad \left| \begin{matrix} C \\ 0 \end{matrix} \right. \rightarrow R$   
 $I_v, I_{uv}$

$$J_o = I_x + I_y = I_u + I_v$$

$I_x, I_y, I_{xy}$

A	$I_x$	B	$I_y$
	$I_{xy}$		$-I_{xy}$



$$R^2 = \left( \frac{I_x - I_y}{2} \right)^2 + I_{xy}^2 \quad \left| \begin{matrix} C = \frac{I_x + I_y}{2} \\ 0 \end{matrix} \right.$$

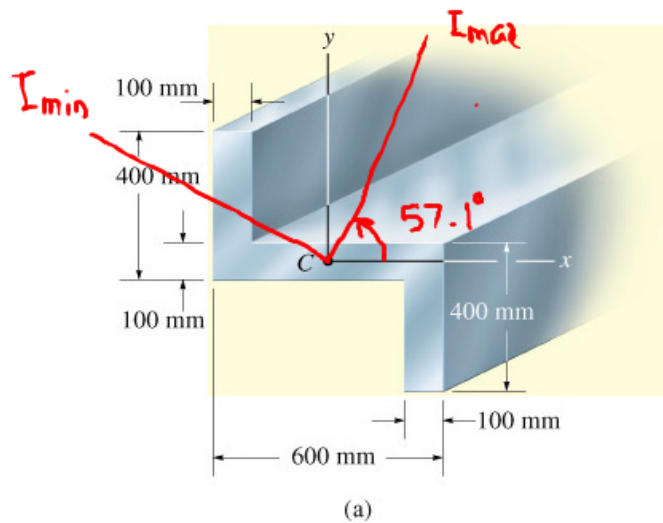
$$\left. \begin{aligned} I_{max} &= \frac{I_x + I_y}{2} + R \\ I_{min} &= \frac{I_x + I_y}{2} - R \end{aligned} \right\} \rightarrow I_{uv} = 0$$

هر محور تفاوتی بین یک محور اصلی  
لختی سطح می باشد.

$$I_{uv} = \frac{I_x - I_y}{2} \sin 2\theta + I_{xy} \cos 2\theta = 0$$

$$\tan 2\theta = \frac{-2I_{xy}}{I_x - I_y}$$

$$\left\{ \begin{aligned} 2\theta &= \tan^{-1} \left( \frac{-2I_{xy}}{I_x - I_y} \right) \\ 2\theta &= \tan^{-1} \left( \frac{-2I_{xy}}{I_x - I_y} \right) + 180^\circ \end{aligned} \right.$$



$$I_x = 2.9 \times 10^9 \text{ mm}^4$$

$$I_y = 5.6 \times 10^9 \text{ mm}^4$$

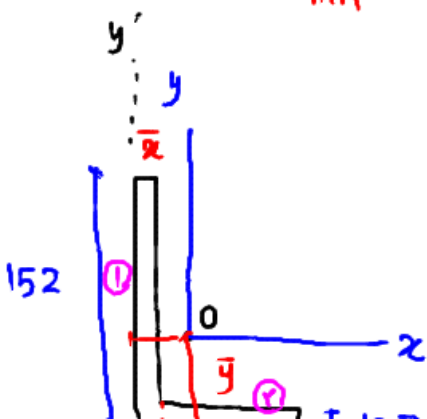
$$I_{xy} = -3 \times 10^9 \text{ mm}^4$$

$$A \begin{vmatrix} I_x \\ I_{xy} \end{vmatrix} \quad B \begin{vmatrix} I_y \\ -I_{xy} \end{vmatrix}$$

مثال:

$$I_{max} = 7.58 \times 10^9 \text{ mm}^4$$

$$I_{min} = 10^9 \text{ mm}^4$$

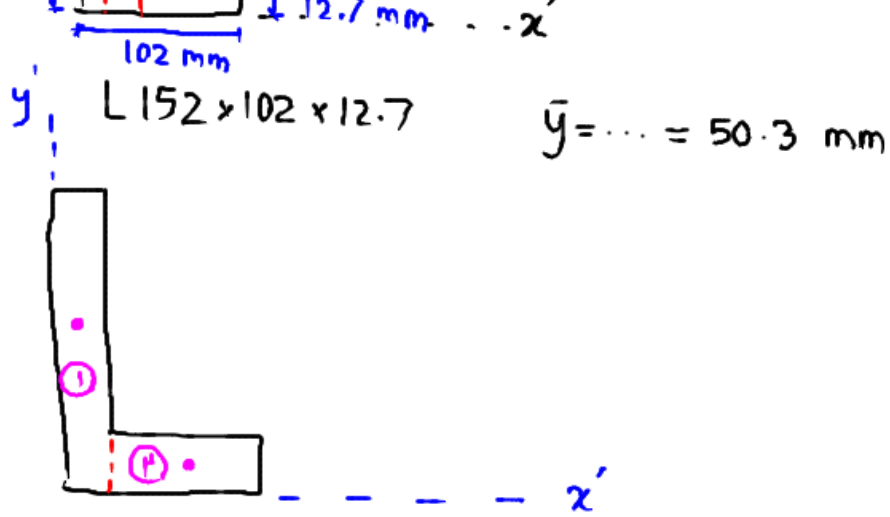


$$\bar{x} = \frac{\int \bar{x} dA}{\int dA} = \frac{\sum \bar{x}_i A_i}{\sum A_i}$$

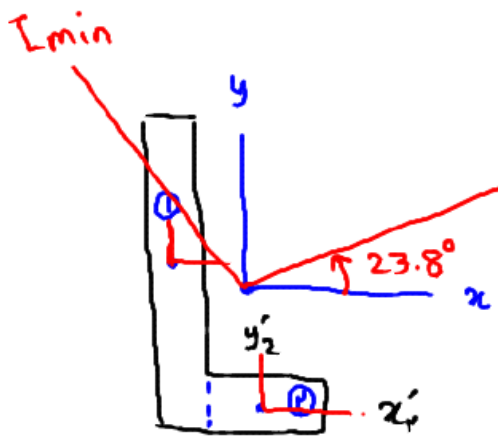
① ماسه مرکز سطح  
خطات

$$\bar{x} = \frac{(152 \times 12.7) \times \frac{12.7}{2} + (89.3 \times 12.7) \times 57.35}{152 \times 12.7 + 89.3 \times 12.7} = 25.3$$

mm



$$\left\{ \begin{array}{l} \bar{x}_1 = \frac{12.7}{2} \\ \bar{y}_1 = \frac{152}{2} = 76 \end{array} \right. \quad \left\{ \begin{array}{l} \bar{x}_2 = \frac{89.3}{2} + 12.7 = 57.35 \\ \bar{y}_2 = \frac{12.7}{2} = 6.35 \end{array} \right.$$



$\bar{I}_{xy}$ ,  $\bar{I}_y$ ,  $\bar{I}_x$       ②

①  $\bar{I}_x = \frac{12.7 \times 152^3}{12} + 25.7^2 \times (12.7 \times 152)$

②  $\bar{I}_x = \frac{12.7^3 \times 89.3}{12} + 43.95^2 \times (12.7 \times 89.3)$

$\therefore \bar{I}_x = \bar{I}_x + \bar{I}_x = 7.2 \times 10^6 \text{ mm}^4$

$\bar{I}_y = 2.64 \times 10^6 \text{ mm}^4$

①  $\bar{I}_{xy} = (-18.95) \left( \frac{25.7}{\bar{y}_1} \right) (12.7 \times 152)$

②  $\bar{I}_{xy} = 32.05 \times (-43.95) (12.7 \times 89.3)$

$\bar{I}_{xy} = \sum \bar{I}_{xy_i} = -2.5 \times 10^6 \text{ mm}^4$

A |  $\bar{I}_x = 7.2 \times 10^6$       B |  $\bar{I}_y = 2.64 \times 10^6$

$$\left. \begin{aligned} \bar{I}_{xy} &= -2.5 \times 10^6 \\ -\bar{I}_{xy} &= 2.5 \times 10^6 \end{aligned} \right\}$$

